4.2 What do I need to know about logarithmic and exponential functions?

2014-11-12 day 53

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Exponential "Rules"

core idea: the exponent tells us "how many times to multiply the base by itself"

This is a sufficient understanding if you are talking about positive integers only

$a^m \cdot a^n = a^{m+n}$

$a^0 = 1$

$a^3 = a \cdot a \cdot a$

$a^2 = a \cdot a = \frac{a^3}{a}$

$a^1 = a = \frac{a^2}{a}$

$a^m \cdot a^{-m} = a^0 = 1 \quad a^0 = 1 = \frac{a}{a}$

So $a^{-m} = \frac{1}{a^m}$

$a^1 = a$

$\frac{a^m}{a^n} = a^{m-n}$

corollary: $\frac{1}{a^{-m}} = \frac{a^m}{1} = a^m$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$[a^{\frac{1}{n}} \cdot a^{\frac{1}{n}} \cdot \ldots \cdot a^{\frac{1}{n}} = a^1]$ (n of them)

does not exist if $a<0$ and n even

$(ab)^m = a^m b^m$

Notice: $(a^m)^n = a^{mn}$

$a(b+c) = ab+ac$

"distributive property of multiplication over addition"

$(ab)^m = a^m b^m$

"the distributive property of exponentiation over multiplication"

Order of Operations

Grouping

Exponentiation

Multiply & Divide

Add & Subtract

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Logarithms

core idea: a logarithm is an exponent

roots	exponential	logarithmic
$\sqrt[b]{c} = a$	$a^b = c$	$\log_a c = b$

very important

logarithm 'rules'

$\log_a(bc) = \log_a b + \log_a c$

$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

$\log_a(b^m) = m \log_a b$

log & exp are inverses

$a^{\log_a b} = b$

$\log_a a^b = b$

the change of base heresy

$\log x = \log_{10} x$

$\ln x = \log_e x$

$x = \log_a b \iff a^x = b$

$\ln(a^x) = \ln(b)$

$x \ln a = \ln b$

$x = \frac{\ln b}{\ln a}$

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3.8/17) local linear approximation

approx complexity w/ lines [tangent line]

a) approx. $\sin x$ near $x_0 = 0$

$y - y_0 = m(x - x_0)$ Pt: (x_0, y_0) $= (0, \sin(0)) = (0,0)$

$m = \frac{d}{dx}(\sin x)\Big|_{x=0}$

$= \cos x\Big|_{x=0} = \cos 0$

$= 1$

$y - 0 = 1(x - 0)$

$y = x$ → linear approx: $y \approx x$

approx [w.r.t.] $\sin 1°$:

$\sin 1°\left(\frac{\pi}{180}\right) = \sin\left(\frac{\pi}{180}\right)$

$\sin\left(\frac{\pi}{180}\right) \approx \frac{\pi}{180}$

$\sin 1° = \sin\frac{\pi}{180} = .017452406$

$\frac{\pi}{180} = .017453293$

c) $44°$

$\sin 44° = \sin 44\left(\frac{\pi}{180}\right)$

approx: $y \approx x$

$\sin 44° \approx \frac{44\pi}{180}$

.694658... ↙ ↘ .767945

c-alternative) $44°$ is real close to $45°$

$y - y_0 = m(x - x_0)$ Pt: $\left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right)$

$= \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right) \approx \left(\frac{\pi}{4}, .707\right)$

$m = \cos(45°) = \frac{\sqrt{2}}{2}$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$\sin 44° = \sin\frac{44\pi}{180} \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(\frac{44\pi}{180} - \frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(\frac{44\pi}{180} - \frac{45\pi}{180}\right)$

$-\frac{\pi}{180}$

$\approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(-\frac{\pi}{180}\right)$

$\approx .707 + .707\left(-\frac{1}{60}\right)$

$\approx .707 - .012 = .695$

60 | .707 (.012, 107, ...)